AP Calculus AB

Tangent and Normal Lines Practice

For #1 and 2:

a) Sketch the graph of the function

b) Find the slope of the tangent line at the given point.

c) Find the equation of the tangent line at the given point. Sketch and label the line.

d) Find the equation of the normal line at the given point. Sketch and label the line.

1. \( f(x) = 6 - x^2, \quad (2, 2) \)

2. \( y = \sqrt{x}, \quad (4, 2) \)
Find the equations of the tangent and normal lines to the curve at the given x-values.

3. \( y = x^2(3 - x), \ x = -2 \)

4. \( y = x - \sqrt{x}, \ x = 4 \)

5. Recall: horizontal lines have a slope \( m = 0 \).
   For what x-values does the curve \( f(x) = \frac{1}{3} x^3 - x^2 \) have horizontal tangent lines?
   *Hint: This means what x-values have tangent lines with a slope of zero.*

6. Find the equation of the tangent line to the curve \( g(x) = x^2 + 6 \) that is parallel to the line \( y = 1 + 3x \).
   *Hint: Find \( g'(x) \) and set it equal to the slope of the line given. Solve for \( x \).
   This is the x-value where the slope of the tangent line is the same as the slope of the line given.
   Plug this x-value in the original function to find the y-value. Use this information to write equation of tangent line.*
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For #1 and 2:

a) Sketch the graph of the function

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c) Find the equation of the tangent line at the given point. Sketch and label the line.

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1. \( f(x) = 6 - x^2 \), (2, 2)

a) 

b) \( f'(x) = -2x \)
\( f'(2) = -4 \)

c) \( 2 = -f'(2) + b \)
\( 2 = -(-4) + b \)
\( 2 = 4 + b \)
\( 10 = b \)
\( y = -4x + 10 \)

d) \( 2 = \frac{1}{4} (2) + b \)
\( 2 = \frac{1}{2} + b \)
\( \frac{3}{2} = b \)
\( y = \frac{1}{4}x + \frac{3}{2} \)

2. \( y = \sqrt{x} \), (4, 2)

a) 

b) \( y' = \frac{1}{2\sqrt{x}} \)
\( y'(4) = \frac{1}{4} \)

c) \( 2 = 4(\frac{1}{4}) + b \)
\( 2 = 1 + b \)
\( b = 1 \)
\( y = \frac{1}{4}x + 1 \)

d) \( 2 = -4(4) + b \)
\( b = 18 \)
\( y = -4x + 18 \)
Find the equations of the tangent and normal lines to the curve at the given x-values.

3. \( y = x^2(3 - x), \; x = -2 \)
   \[ y = 3x^2 - x^3 \]
   \[ y' = 6x - 3x^2 \]
   \[ y'(-2) = 6(-2) - 3(-2)^2 = -24 \]
   \[ y(-2) = (-2)^2(3 - (-2)) = 4(5) = 20 \]

4. \( y = x - \sqrt{x}, \; x = 4 \)
   \[ y' = 1 - \frac{1}{2x^{1/2}} \]
   \[ y'(4) = 1 - \frac{1}{2} = \frac{1}{2} \]
   \[ y(4) = 4 - \sqrt{4} = 2 \]

5. Recall: horizontal lines have a slope \( m=0 \).
   For what \( x \)-values does the curve \( f(x) = \frac{1}{3}x^3 - x^2 \) have horizontal tangent lines?
   *Hint:* This means what \( x \)-values have tangent lines with a slope of zero.
   \[ f'(x) = x^2 - 2x = 0 \]
   \[ x(x-2) = 0 \]
   \[ x = 0, \; x = 2 \]
   At \( x = 0 \) and \( x = 2 \), the curve has horizontal tangent lines.

6. Find the equation of the tangent line to the curve \( g(x) = x^2 + 6 \) that is parallel to the line \( y = 1 + 3x \).
   *Hint:* Find \( g'(x) \) and set it equal to the slope of the line given. Solve for \( x \).

This is the \( x \)-value where the slope of the tangent line is the same as the slope of the line given.
Plug this \( x \)-value in the original function to find the \( y \)-value. Use this information to write equation of tangent line.

\[ g'(x) = 2x = 3 \]
\[ x = \frac{3}{2} \]
\[ g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 6 \]
\[ = \frac{9}{4} + \frac{24}{4} = \frac{33}{4} \]
\[ P = \left(\frac{3}{2}, \frac{33}{4}\right) \]